

Trees That Grow

(an early draft – feedback is sought)

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Abstract: We study the notion of extensibility in functional data types, as a new approach to the problem of decorating abstract syntax trees with additional sets of information. We observed the need for such extensibility while redesigning the data types representing Haskell abstract syntax inside GHC.

Specifically, we describe our approach to the tree-decoration problem using a novel syntactic machinery in Haskell for expressing extensible data types. We show that the syntactic machinery is complete in that it can express all the syntactically possible forms of extensions to algebraic data type declarations. Then, we describe an encoding of the syntactic machinery based on the existing features in Glasgow Haskell Compiler (GHC).

1 Introduction

Algebraic Data Types (ADTs) and pattern matching in functional languages lay a fertile ground to conveniently define and process data types as tree-like structures. However, in these grounds, trees often cannot grow; once a data type is defined and compiled, its definition cannot be extended. A data type can be extended, for instance, by adding new data constructors, and/or by adding new fields to its existing data constructors.

At the centre of all compilers stand tall trees representing the abstract syntax of terms. Compiler programs processing these trees often do so by decorating the trees with additional information. For instance, name resolution phase adds information about names, and type inference phase stores the inferred types in the relevant nodes. We refer to such extra information as decorations. The additional information may appear, for instance, as new fields to the existing data constructors, and/or new data constructors in data types representing the trees.

Common practice in compilers is either to define a new separate data type representing the output decorated trees, or to use the same large data type to represent both the non-decorated input and the decorated output trees. Both

methods are unsatisfactory: the former leads to duplication, and the latter forces the input trees to carry an unnecessary set of information making them inconvenient to work with.

We propose a third approach: declare abstract syntax trees with extensible data types, and view decorations in trees as sets of extensions to the data type declarations. Extensible data types are the soil in which trees can grow; extensible data types allow for an arbitrary set of extensions to the same parametric data type declaration, even after they are compiled. Our proposed approach avoids duplication, since the same base declaration is reused for both non-decorated input trees and the decorated output trees. Since non-decorated trees are declared by the base (non-extended) data type declarations, there is no unnecessary set of information baked into the input trees, making them convenient to work with. Section 2 demonstrates the problem and our approach with a running example.

Our proposed approach relies on extensible data types which are less commonly supported, and often missing as an off-the-shelf feature in functional languages like Haskell. To be able to adopt this approach in a language like Haskell, we need to go back to the drawing board, and study the notion of extensibility for data types in such languages.

Extensions to a data type declaration can appear in different forms. Earlier, we enumerated two forms as examples: new fields to the existing data constructors, and/or new data constructors. We can also consider extensions to the set of type parameters in a declaration, or in a setting supporting existential types, we may as well consider extensions to the set of existentially quantified type variables in the existing data constructors. For a systematic, yet simple, study, we consider all the syntactically possible forms of extensions to a generalised algebraic data type (ADT) declaration. See Section 3 for more details about this study.

Having identified different forms of extensions, we describe simple encodings of extensible data types, within Glasgow Haskell Compiler (GHC), allowing all the identified forms of extensions by instantiating the same parametric declaration. In our encodings, extensible data types are parameterised by a set of parameters representing different forms of extensions, and the act of extending a data type declaration is simply instantiating these parameters. Setting the same parameters to the corresponding base cases (e.g., to a type similar to empty type *Void*, monoidal zero of sum types, or to unit type $()$, monoidal zero of product types), yields declaration of non-decorated trees. In Section 4, we explain the details of such encodings.

2 Decorating Trees

In this section, we demonstrate the problem with decorating trees and explain our solution with a running example.

2.1 Tree-Decoration Problem

Consider the following language of lambda terms with integer literals, explicit type annotations (simple types), tuples (pairs), and let expressions with simple variable bindings and tuple pattern bindings (projections).

$$\begin{aligned}
 i &\in \text{integers} \\
 x, y &\in \text{variables} \\
 A, B, C &\in \text{Typ} ::= \mathbf{Int} \mid A \rightarrow B \mid A \times B \\
 L, M, N &\in \text{Exp} ::= i \mid x \mid M :: A \mid \lambda x. N \mid L M \mid (M, N) \mid \mathbf{let } D \mathbf{ in } N \\
 D &\in \text{Dec} ::= x := M \mid (x, y) := L
 \end{aligned}$$

In Haskell, the language above can be declared as the following ADT.

```

type Var = String
data Typ = Int | Typ -> Typ | Typ *: Typ
data Exp = Lit Integer | Var Var | Typ Exp Typ | Abs Var Exp
          | App Exp Exp | Tup Exp Exp | Let Dec Exp
data Dec = Val Var Exp | Prj Var Var Exp

```

Defining a simple printer for this data type is straightforward:

```

printT :: Typ -> String
printT Int      = "Int"
printT (a -> b)  = "(" ++ printT a ++ " " ++ printT b
printT (a *: b)  = "(" ++ printT a ++ " " ++ printT b

printE :: Exp -> String
printE (Lit i)   = show i
printE (Var x)   = x
printE (Typ m a) = "(" ++ printE m ++ " " ++ printT a ++ ")"
printE (Abs x n) = "\ " ++ x ++ ". " ++ printE n
printE (App l m) = "(" ++ printE l ++ " " ++ printE m ++ ")"
printE (Tup m n) = "(" ++ printE m ++ " , " ++ printE n ++ ")"
printE (Let d n) = "let " ++ printD d ++ " in " ++ printE n

printD :: Dec -> String
printD (Val x m) = x ++ " := " ++ printE m
printD (Prj x y l) = "(" ++ x ++ " , " ++ y ++ " ) := " ++ printE l

```

Now consider the following standard type system for the above language,

with Γ and Δ ranging over standard type environments.

$$\begin{array}{c}
\boxed{\Gamma \vdash M : A} \quad \frac{(x : A) \in \Gamma}{\Gamma \vdash x : A} \quad \frac{\Gamma \vdash M : A}{\Gamma \vdash M :: A : A} \quad \frac{\Gamma \vdash M : A \quad \Gamma \vdash N : B}{\Gamma \vdash (M, N) : A \times B} \\
\\
\frac{x : A, \Gamma \vdash N : B}{\Gamma \vdash \lambda x. N : A \rightarrow B} \quad \frac{\Gamma \vdash L : A \rightarrow B \quad \Gamma \vdash M : A}{\Gamma \vdash L M : B} \quad \frac{\Gamma \vdash D \rightsquigarrow \Delta \quad \Delta, \Gamma \vdash N : A}{\Gamma \vdash \text{let } D \text{ in } N : A} \\
\\
\boxed{\Gamma \vdash D \rightsquigarrow \Delta} \quad \frac{\Gamma \vdash M : A}{\Gamma \vdash x := M \rightsquigarrow [x : A]} \quad \frac{\Gamma \vdash L : A \times B}{\Gamma \vdash (x, y) := L \rightsquigarrow [x : A, y : B]}
\end{array}$$

Before type checking, often abstract syntax trees (ASTs) are processed by a type inference engine. The output of the type inference engine is the same input tree decorated with additional type information. Type inference helps users to leave certain bits of their programs without explicit type annotations. Type inference also helps in simplifying the type checker: after type inference, and decorating the trees with the additional type information, type checking becomes a straightforward syntax-directed recursive definition. To accommodate for the additional information in the output, we need larger trees, and hence we need to *extend* the original declarations. For instance, the following highlights the required changes to the *Exp* data type (besides the trivial updates of *Dec* to *Dec*[•] and *Exp* to *Exp*[•]).

```

type TypEnv = [(Var, Typ)]
data Exp• = Lit• Integer | Var• Var | Typ• Exp• Typ
           | Abs• Var Exp• | App• Typ Exp• Exp•
           | Tup• Exp• Exp• | Let• TypEnv Dec• Exp•
data Dec• = Val• Var Exp• | Prj• Var Var Exp•

```

Thanks to this update, type checking is a straightforward structural recursive definition:

```

deriving instance Eq Typ
chkE :: Exp• → TypEnv → Typ → Bool
chkE (Lit• _) _ Int = True
chkE (Var• x) _ Γ c = maybe False (≡ c) (lookup x Γ)
chkE (Typ• m a) Γ c = a ≡ c ∧ chkE m Γ c
chkE (Abs• x n) Γ (a → b) = chkE n ((x, a) : Γ) b
chkE (App• a l m) Γ c = chkE l Γ (a → c) ∧ chkE m Γ a
chkE (Tup• m n) Γ (a × b) = chkE m Γ a ∧ chkE n Γ b
chkE (Let• Δ d n) Γ c = chkD d Γ Δ ∧ chkE n (Δ ++ Γ) c
chkE _ _ _ = False
chkD :: Dec• → TypEnv → TypEnv → Bool
chkD (Val• x m) Γ [(x', a)] = x ≡ x' ∧ chkE m Γ a

```

$$\begin{array}{lcl}
chkD \ (Prj^\bullet \ x \ y \ l) \ \Gamma \ [(x', a), (y', b)] & = & x \equiv x' \wedge y \equiv y' \wedge chkE \ l \ \Gamma \ (a \text{ :}^* b) \\
chkD \ _ & \quad \quad \quad & = False
\end{array}$$

To use the printer defined earlier with the new decorated trees, the definition should be updated so that it ignores the additional information:

$$\begin{array}{lcl}
printE^\bullet :: Exp^\bullet \rightarrow String \\
printE^\bullet \ (App^\bullet \ _ \ l \ m) & = & \dots \\
printE^\bullet \ (Let^\bullet \ _ \ d \ n) & = & \dots \\
printE^\bullet \ \dots & = & \dots \\
printD^\bullet :: Dec^\bullet \rightarrow String \\
printD^\bullet \ \dots & = & \dots
\end{array}$$

To recap, so far we have defined

1. the non-decorated trees using *Exp* and *Dec* (and *Typ*);
2. the decorated trees using *Exp*[•] and *Dec*[•];
3. the printer for the non-decorated trees using *printE* and *printD* (and *printT*);
4. the printer for the decorated trees using *printE*[•] and *printD*[•]; and
5. the type checker for the decorated trees using *chkE* and *chkD*.

What about the set of functions in the parser generating these trees, or the type inference engine applied before type checking? What should be their types?

Common practice is either to use the decorated variants (e.g., *Exp*[•]) when the additional information in decorations is needed, and use the non-decorated variants (e.g., *Exp*) otherwise (when possible); or, use the decorated variants everywhere. For instance, with the former setting, they will be

$$\begin{array}{lcl}
parseE :: String \rightarrow Maybe \ Exp \\
parseD :: String \rightarrow Maybe \ Dec \\
inferE :: Exp \rightarrow Exp^\bullet \\
inferD :: Dec \rightarrow Dec^\bullet
\end{array}$$

and with the later setting, they will be

$$\begin{array}{lcl}
parseE^\bullet :: String \rightarrow Maybe \ Exp^\bullet \\
parseD^\bullet :: String \rightarrow Maybe \ Dec^\bullet \\
inferE^\bullet :: Exp^\bullet \rightarrow Exp^\bullet \\
inferD^\bullet :: Dec^\bullet \rightarrow Dec^\bullet
\end{array}$$

The former leads to duplication, both in the definition of data types and in the functions defined over them. While for the latter, there is no need for declarations

of *Exp* and *Dec*, and the corresponding set of functions whose equivalents are available for *Exp*[•] (e.g., *printE* and *printD*).

On the other hand, for the latter, all functions using the decorated trees (the only available variant then) should deal with the decorations explicitly, even if their functionality is entirely independent of the decorations. This entanglement is harmful: the more passes processing the trees, the larger the set of unnecessary decorations. Furthermore, the decorations introduce unnecessary dependencies between parts that define the decorations and parts that are forced to depend on them since decorations are baked into their input trees (and they do not use them). Notice, not every function can ignore the unnecessary annotations, like *printE*[•] could; some, specially tree-to-tree transformations, have to push the decorations around without looking at them, or even worst, to generate dummy decorations to be able to apply a data constructor with decorations.

2.2 Tree-Decoration Solution

As mentioned earlier, we suggest declaring ASTs with extensible data types, and view decorations in trees as sets of extensions to the data type declarations.

To avoid the need for explaining the details of encodings of extensible data types prematurely, we explain our solution using an idealised syntax (i.e., a macro) that we developed for Haskell. This allows our solution to be independent of the implementation details (e.g., the encodings). This syntax allows us to declare extensible data types, by labeling normal algebraic data type declarations as extensible, and allows us to define extensions to an extensible algebraic data type declaration by specifying

- (a) new fields to the existing data constructors of the extensible data type,
- (b) new data constructors to the extensible data type, and
- (c) new type parameters (with alpha renaming of the existing ones, if needed).

As an example, consider the definition of *Exp* from earlier, without bits related to tuples:

$$\begin{array}{ll}
 i & \in \text{integers} \\
 x, y & \in \text{variables} \\
 A, B, C & \in \text{Typ} ::= \mathbf{Int} \mid A \rightarrow B \\
 L, M, N & \in \text{Exp} ::= i \mid x \mid M :: A \mid \lambda x. N \mid L M \mid \mathbf{let } D \mathbf{ in } N \\
 D & \in \text{Dec} ::= x := M
 \end{array}$$

An extensible declaration of the above language is as follows.

extensible data Exp_X
 $= Lit_X \ Integer$
 $| Var_X \ Var$
 $| Typ_X \ Exp_X \ Typ_X$
 $| Abs_X \ Var \ Exp_X$
 $| App_X \ Exp_X \ Exp_X$
 $| Let_X \ Dec_X \ Exp_X$

extensible data Typ_X
 $= Int_X$
 $| Typ_X \overset{x}{\rightarrow} Typ_X$
extensible data Dec_X
 $= Val_X \ Var \ Exp_X$

To define a datatype as extensible, we just add the label **extensible** before a normal ADT declaration.

Defining printer for the extensible data type Exp_X above is similar to the one for Exp^\bullet in that it ignores the decorations. However, in Exp_X , decorations are not bound to be of a specific type: they are polymorphic, instantiated explicitly by the extensions.

$printE_X (Lit_X \ i \oplus _) = \dots$ $printT_X (Int_X \oplus _) = \dots$
 $printE_X (Var_X \ x \oplus _) = \dots$ $printT_X (a \overset{x}{\rightarrow} b \oplus _) = \dots$
 $printE_X (Typ_X \ m \ a \oplus _) = \dots$ $printT_X (Typ_X \oplus e) = ?printT_{Ext} \ e$
 $printE_X (Abs_X \ x \ n \oplus _) = \dots$ $printD_X (Val_X \ x \ m \oplus _) = \dots$
 $printE_X (App_X \ l \ m \oplus _) = \dots$ $printD_X (Dec_X \oplus e) = ?printD_{Ext} \ e$
 $printE_X (Let_X \ d \ n \oplus _) = \dots$
 $printE_X (Exp_X \oplus e) = ?printE_{Ext} \ e$

The pattern syntax $C_X \ P_1 \dots P_n \oplus P'$ describes matching on the existing fields of the constructor C_X of an extensible datatype by patterns P_1 to P_n and matching on the new field (introduced via extensions) to that constructor by patterns P' . The pattern syntax $P' \oplus T_X$ describes matching on the new constructors (introduced via extensions) to the extensible data type T_X by pattern P' .

In the above example, to avoid cluttering the presentation with explicit passing of functions, we have used implicit parameters to introduce functions that are applied to the extensions. We could as well use methods of a type class (say named *Printable*), or normal explicit passing of functions.

We can use a similar syntax to construct values of an extensible data type. For instance, consider the following.

data $SrcSpan = SrcSpan \{ begins :: Int, ends :: Int \}$
 $myLit = Lit_X \ 42 \oplus (SrcSpan \ 0 \ 2)$

It is meant to construct an integer literal value of Exp_X , with an extension field describing its position in a source file. Looking at its inferred type is instructional: the inferred type is $(\xi \ "LitX" \sim SrcSpan) \Rightarrow Exp_X \oplus \xi$. The inferred type reads as “there is an extension named ξ to Exp_X that for the constructor

Lit_X (i.e., the new field introduced by the extension ξ in Lit_X) it is of the type $SrcSpan$ ".

Unfortunately, a type of this form is not currently accepted by GHC: to keep the type-level machinery consistent with the type inference in Haskell, type functions (i.e., type families) in GHC are not first class, and they cannot be quantified over at the type-level. This means ξ should be a concrete type constructor. We can, for instance choose ξ to be the constant functor $Const SrcSpan$, setting the type of all new fields (and constructor) introduced by the extension to be $SrcSpan$. That is, we can write $Lit_X \ 42 \oplus (Const (SrcSpan \ 0 \ 2))$ instead, whose inferred type is $Exp_X \oplus (Const SrcSpan)$. To have a more refined control over the type of each extension, and to avoid problems with type inference, in what follows we describe a way to declare extensions. Declaring extensions, as opposed to having them inferred from the context (or by complex type annotations) is synonymous to having declared records as opposed to anonymous records; declared constructors as opposed to anonymous variants; or iso-recursive types, as opposed to equi-recursive types.

Let us start with an example. The extension that introduces the bits related to tuples that we dropped earlier (the extension form (b) above), and that also introduces the inferred type decorations discussed earlier (the extension form (a) above) is defined as follows.

<pre> type $TypEnv_X^\bullet = [(Var, Typ_X^\bullet)]$ data Exp_X^\bullet extends Exp_X = $Tup_X^\bullet Exp_X^\bullet Exp_X^\bullet$ Lit_X^\bullet extends Lit_X by \emptyset Var_X^\bullet extends Var_X by \emptyset Typ_X^\bullet extends Typ_X by \emptyset Abs_X^\bullet extends Abs_X by \emptyset App_X^\bullet extends App_X by Typ_X^\bullet Let_X^\bullet extends Let_X by $TypEnv_X^\bullet$ </pre>	<pre> data Typ_X^\bullet extends Typ_X = $Typ_X^\bullet \text{ : } Typ_X^\bullet$ Int_X^\bullet extends Int_X by \emptyset $(\overset{x}{\rightarrow})^\bullet$ extends $(\overset{x}{\rightarrow})$ by \emptyset data Dec_X^\bullet extends Dec_X = $Prj_X^\bullet Var Var Exp_X^\bullet$ Val_X^\bullet extends Val_X by \emptyset </pre>
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The syntax **data** $T \ \alpha_1 \dots \alpha_n$ **extends** $T_X \ \beta_1 \dots \beta_m = \dots$ declares T with type variables a_1 to a_n , as an extension of T_X where type variables of T_X in the position 1 to m are set to variables b_1 to b_m . A variables b_i should be one of the variables a_1 to a_n (not necessarily the same order), so we have: $b_i \in \{a_1 \dots a_n\}$ and $m \leq n$. The declarations allows for extension of form (c) mentioned earlier, where an extended data type may have an additional set of variables compared to the base extensible data type. We refer to T as the extending data type, and T_X as the base data type.

The syntax C **extends** C_X **by** $T_1 \dots T_n$ declares the constructor C for the extending data type that has all the fields of the constructor C_X of the base

data type in addition to the fields of the types T_1 to T_n .

The constructor declarations without **extends...by...** are the new constructors that the extending data type extends the base data type with.

The data types Typ_X^\bullet , Exp_X^\bullet , and Dec_X^\bullet are equivalent to Typ^\bullet , Exp^\bullet , and Dec^\bullet in that they have data constructors of the same signature and can carry the same information (i.e., isomorphic). The type checking function for Exp_X^\bullet and Dec_X^\bullet would be exactly the same as the ones for Exp^\bullet and Dec^\bullet . Same applies for the printer function. Yet, thanks to extensibility, the extending data types can reuse the one defined for the base data type.

$$\begin{aligned}
& printT_{Ext}^\bullet :: Typ_X^\bullet \sim Typ_X \oplus \xi \Rightarrow \xi \text{ "TypX" } \rightarrow String \\
& printT_{Ext}^\bullet (_ \oplus a \text{ : } b) = "(" ++ printT_X^\bullet a ++ ")" ++ printT_X^\bullet b \\
& printE_{Ext}^\bullet :: Exp_X^\bullet \sim Exp_X \oplus \xi \Rightarrow \xi \text{ "ExpX" } \rightarrow String \\
& printE_{Ext}^\bullet (_ \oplus Tup_X^\bullet m n) = "(" ++ printE_X^\bullet m ++ " , " ++ printE_X^\bullet n ++ ")" \\
& printE_{Ext}^\bullet :: Dec_X^\bullet \sim Dec_X \oplus \xi \Rightarrow \xi \text{ "DecX" } \rightarrow String \\
& printD_{Ext}^\bullet (_ \oplus Proj_X^\bullet x y l) = "(" ++ x ++ " , " ++ y ++ ")" := " ++ printE_X^\bullet l \\
& printT_X^\bullet :: Typ_X^\bullet \rightarrow String \\
& printT_X^\bullet = \text{let } ?printT_{Ext} = printT_{Ext}^\bullet \text{ in } printT_X \\
& printE_X^\bullet :: Exp_X^\bullet \rightarrow String \\
& printE_X^\bullet = \text{let } ?printT_{Ext} = printT_{Ext}^\bullet \\
& \quad ?printE_{Ext} = printE_{Ext}^\bullet \\
& \quad ?printD_{Ext} = printD_{Ext}^\bullet \\
& \quad \text{in } printE_X \\
& printD_X^\bullet :: Dec_X^\bullet \rightarrow String \\
& printD_X^\bullet = \text{let } ?printT_{Ext} = printT_{Ext}^\bullet \\
& \quad ?printE_{Ext} = printE_{Ext}^\bullet \\
& \quad ?printD_{Ext} = printD_{Ext}^\bullet \\
& \quad \text{in } printD_X
\end{aligned}$$

The pattern syntax $_ \oplus C P_1 \dots P_n$ describes matching only on the extensions (e.g., the decorations) in the constructor K of an extending data type by patterns P_1 to P_n . The type $T \sim T_X \oplus \xi \Rightarrow \xi \text{ "S"}$ can be read as “ ξ is the extension by which T extends T_X , and we are interested in the specific extension at S , where S ranges over the names of the constructors in T_X (and data types defined mutually-recursively with it) to represent new fields to that constructor, and the name T_X itself (and data types defined mutually-recursively with it) to represent the set of new constructors . We could as well introduce the notation $(T \ominus T_X) \text{ "S"}$ to express the same.

Notice how we managed to define the printer in a compositional way. However, in general, it may not be practically possible to define functions over an extending data type as a composition of functions defined separately over the base data type and the pieces of extensions. Yet still, extensible data types allow for reuse of data type declarations even if functions defined over them cannot be reused.

Following the same techniques, we can reuse the base data type delcarations

to define data types equivalent to the ones representing the non-decorated AST, i.e., Typ , Exp , and Dec :

<pre> type $TypEnv_X^\circ = \dots$ data Exp_X° extends Exp_X = ... App_X° extends App_X by \emptyset Let_X° extends Let_X by \emptyset </pre>	<pre> data Typ_X° extends Typ_X = ... data Dec_X° extends Dec_X = ... </pre>
---	--

Once again, we can define the printer function by providing a printer for the pieces of extension, and it is the same as the one for Typ_X^\bullet , Exp_X^\bullet , and Dec_X^\bullet .

To recap, so far (in this subsection) we have defined

1. the extensible trees using Typ_X , Exp_X , and Dec_X ;
2. the non-decorated trees as an extension to the extensible trees using the extending data types Typ_X° , Exp_X° , and Dec_X° ;
3. the decorated trees as an extension to the extensible trees using the extending data types Typ_X^\bullet , Exp_X^\bullet and Dec_X^\bullet ;
4. the printer for the extensible trees using $printT_X$, $printE_X$, and $printD_X$;
5. the printer for the non-decorated trees by only defining printers for the extensions using $printT_X^\circ$, $printE_X^\circ$, and $printD_X^\circ$;
6. the printer for the decorated trees by only defining printers for the extensions using $printT_X^\bullet$, $printE_X^\bullet$, and $printD_X^\bullet$;
7. the type checker for the decorated trees using $chkE_X$ and $chkD_X$.

In this extensible setting, the types of the set of functions in the parser and in the type inference engine are

```

 $parseT_X :: String \rightarrow Maybe\ Typ_X^\circ$ 
 $parseE_X :: String \rightarrow Maybe\ Exp_X^\circ$ 
 $parseD_X :: String \rightarrow Maybe\ Dec_X^\circ$ 

 $inferE_X :: Exp_X^\circ \rightarrow Exp_X^\bullet$ 
 $inferD_X :: Dec_X^\circ \rightarrow Dec_X^\bullet$ 

```

Extensibility has bought us reusability of data type declarations (and reusability of functions in the case of printers), and also it has brought us modularity of the definitions, in that the definitions of the decorations are separated from the definition of the trees. Is this all we can do? In the next section, we identify the set of syntactically possible forms of extensions to a data type declaration.

3 All You Can Do

The notion of extension for a set, or for a list, is obvious: you can extend a set by adding one or more elements to the set. What if you are given a pair of two sets, the set A and the set B? There you have two forms of extensions: extension to set A, and/or, extension to set B. Similarly for a set of sets, you can extend the mother set, and/or, extend the existing child sets. However, if you are given a pair of an atomic value (non-extensible) and a set, there will be only one form of extensions: extending the only set. In this section, we study notion of extension to algebraic data type declarations and generalised algebraic data type declarations.

3.1 Extensions in Algebraic Data Types

Syntactically speaking, an algebraic data type declaration (ADT) in Haskell can be seen as the structure

$$(TyConId, \{ VarId \}, \{ (ConId, [Type]) \})$$

where *TyConId* represents the type constructor identifiers in Haskell, *VarID* represents the type variable identifiers, *ConId* represents the data constructor identifiers, *Type* represents the syntax of Haskell types, $\{ _ \}$ denotes sets, and $[_]$ denotes lists. The syntax of ADT declarations, excluding the atomic parts (we noticeably consider types, except the ones defined mutually-recursively to be atomic at this stage), consists of a set (type variables) and a set of lists (constructors). Following a similar reasoning as before, syntactically there are three possible forms of extensions to an ADT declaration:

- (a) extensions to the list of fields of each data constructors,
- (b) extensions to the set of data constructors, and
- (c) extensions to the set of type parameters.

This matches exactly our specification of possible extensions to a data type in the previous section; using the notations introduced in the previous section, we can define extensions to an extensible algebraic data type declaration by specifying

- (a) new fields to the existing data constructors of the extensible data type,
- (b) new data constructors to the extensible data type, and
- (c) new type parameters (with alpha renaming of the existing ones, if needed).

Therefore, our notation for extensible data types is complete with respect to the set of syntactically possible extensions to an algebraic data type declaration.

We can go a few steps further, and consider all syntactically possible forms of extensions to generalised algebraic data type (GADTs) declarations.

3.2 Extensions in Generalised Algebraic Data Types

The syntax of a generalised algebraic data type declaration (GADT) in Haskell (ignoring the kind annotations) can be seen as the structure

$$(TyConId, \{ VarId \}, \{ (ConId, \{ VarId \}, \{ Constraint \}, [Type]) \})$$

where *TyConId*, *VarId*, *ConId*, and *Type* are as before, and *Constraint* represents the syntax of type constraints in Haskell (e.g., type class constraints, or type equality constraints). Following a similar reasoning as before, syntactically there are two additional (compared to ADTs) possible forms of extensions to a GADT declaration:

- (d) extensions to the set of local type variables, and
- (e) extensions to the set of local type constraints.

Although we have not provided a syntax to describe extensions to a GADT declaration, in theory it is straightforward to do so.

4 GHC Can Do

In this section, we present encodings of extensible data types in GHC Haskell. Our encodings allow for all extension forms identified in the previous section for ADTs.

4.1 Extensible Algebraic Data Types

The idea behind our encoding is simple: to make an ADT declaration extensible, introduce additional parameters to stand for possible extensions, and instantiate these parameters per extending data types.

The simplest, yet practically naïve, implementation of such encoding for our running example *Typ_X* would be as follows:

extensible data <i>Typ_X</i> $= Int_X$ $ Typ_X \xrightarrow{x} Typ_X$	\mapsto	data <i>Typ_X</i> <i>xInt_X</i> <i>xArr_X</i> <i>xTyp_X</i> $= Int_X \ xInt_X$ $ (\xrightarrow{x}) \ xArr_X$ $\quad (Typ_X \ xInt_X \ xArr_X \ xTyp_X)$ $\quad (Typ_X \ xInt_X \ xArr_X \ xTyp_X)$ $ Typ_X \ xTyp_X$
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where *xInt_X* stands for new field extensions to the constructor *Int_X*, *xArr_X* stands for new field extensions to the constructor \xrightarrow{x} , and *xTyp_X* to new constructor extensions to *Typ_X*.

We can practically improve above encoding by adding only one (higher-order) parameter, and project the extension parameters by a set of unique labels:

extensible data Typ_X $= Int_X$ $ \quad Typ_X \xrightarrow{x} Typ_X$	\mapsto	data $Typ_X \xi$ $= Int_X \ (\xi \text{ "IntX"})$ $ \quad (\xrightarrow{x}) \ (\xi \text{ "ArrX"}) (Typ_X \xi) (Typ_X \xi)$ $ \quad Typ_X \ (\xi \text{ "TypX"})$
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The data types extending Typ_X can be defined by setting the parameters. However, in the latter, more compact, variant we need to instantiate a higher-order (indexed) parameter in Haskell, and we can choose to do so either by GADTs, or by data families. For example, the extending data type Typ_X^\bullet from earlier can be encoded as follows:

data Typ_X^\bullet extends Typ_X $= Typ_X^\bullet \text{ :* } \bullet Typ_X^\bullet$ $ \quad Int_X^\bullet$ extends Int_X by \emptyset $ \quad (\xrightarrow{x}^\bullet)$ extends (\xrightarrow{x}) by \emptyset	\mapsto	type $Typ_X^\bullet = Typ_X \ Ext$ data family $Ext \ (label :: Symbol) :: *$ data instance $Ext \text{ "TypX"}$ $= Typ_X^\bullet \text{ :* } 'Typ_X^\bullet$ data instance $Ext \text{ "IntX"} = NoneI$ data instance $Ext \text{ "ArrX"} = NoneA$ pattern $m \text{ :* } \bullet n = Typ_X \ (m \text{ :* } 'n)$ pattern $Int_X^\bullet = Int_X \ NoneI$ pattern $m \xrightarrow{x}^\bullet n = (\xrightarrow{x}) \ NoneA \ m \ n$
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The process of translating from our syntax to the underlying encoding is as follows.

Declarations:

extensible data $T' \ \alpha_1 \dots \alpha_n$ $= \dots$ $ \quad C_i \dots T_{i,j} \dots$ $ \quad \dots$	\mapsto	data $T' \ \xi \ \alpha_1 \dots \alpha_n$ $= T' \ (\xi \text{ "T' "})$ $ \quad \dots$ $ \quad C_i \ (\xi \text{ "C}_i") \dots \llbracket T_{i,j} \rrbracket_\xi \dots$ $ \quad \dots$
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$\llbracket T \rrbracket_\xi = T \ \xi$	if T is extensible
$\llbracket T \rrbracket_\xi = T$	if T is not extensible

Types:

$T_K \ T_1 \dots T_n \oplus T_\xi$	\mapsto	$T_K \ T_\xi \ T_1 \dots T_n$
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Patterns:

$C \ P_1 \dots P_n \oplus P'$	\mapsto	$C \ P' \ P_1 \dots P_n$
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Data Constructors:

$C \ M_1 \dots M_n \oplus M'$	\mapsto	$C \ M' \ M_1 \dots M_n$
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Extensions:

<pre> data $T' \dots \alpha_i \dots$ extends ($T \dots \beta_j \dots$) = ... $C'_{i'}$ extends $C_{i'}$ by $T_{i',1} \dots T_{i',m}$... $C'_{j'} T_{j',1} \dots T_{j',n}$... </pre>	\mapsto	<pre> type $T' \dots \alpha_i \dots$ = $T \text{ Ext}^u \dots \beta_j \dots$ data family Ext^u ($\text{label} :: \text{Symbol}$) :: * data instance Ext^u "T" = ... $C'_{j'} T_{j',1} \dots T_{j',n}$ data instance Ext^u "C'_{i'}" = $C'^u_{i'} \dots T_{i',k} \dots$ pattern $C'_{i'} x_1 \dots x_m y_1 \dots y_k$ = $C_{i'} (C'^u_{i'} x_1 \dots x_m) y_1 \dots y_k$... </pre>
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We write Id^u , or id^u , to represent unique generated names. This encoding naturally scales to mutually-recursive definitions by using the same extension data family.

4.2 Extensible Generalised Algebraic Data Types

So far, we have considered encoding of extensible algebraic data type declarations. We can extend our results to generalised algebraic data types.

References